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Engineering Notes

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Entry Guidance Using Analytical Atmospheric Skip Trajectories

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Introduction

ATMOSPHERIC skip-entry trajectories were designed for the Apollo lunar missions [1] and are being considered again for new vehicles with similar low-lift-to-drag L/D ratios [2]. A skip entry begins at entry interface (EI) with hyperbolic or near-hyperbolic speed and shallow flight-path angle, and the aerodynamic lift force is used to continually rotate the velocity vector upward until the vehicle exits the sensible atmosphere. After atmospheric exit, the vehicle follows a Keplerian coast arc until it reenters the atmosphere. Skip-entry trajectories offer the advantage of downrange control by targeting a desired exit state (velocity V_{exit} and flight-path angle γ_{exit}) at atmospheric skip-out.

Bank angle is the single control variable during entry, and rotating the vehicle about its velocity vector modulates the vertical component of the aerodynamic lift force. The Apollo entry guidance system used closed-form trajectory solutions to predict V_{exit} and γ_{exit}, and these analytical equations were developed by applying simplifying approximations to the nonlinear equations of motion [3]. Total downrange distance was computed by piecing together range estimates for the initial atmospheric skip path, the Keplerian coast arc, and the final atmospheric flight after reentry. Bank angle was adjusted until the predicted downrange distance matched the range-to-go.

Although the Apollo entry guidance method proved to be very successful, it had limited range-prediction accuracy (its maximum range capability was set at 2500 n mile, or a great-circle arc of about 41 deg) [3]. Current design goals for a new low-L/D vehicle include extending range capability for abort scenarios and other contingencies. Bairstow [4] and Brunner and Lu [5] both developed numerical predictor–corrector guidance methods that propagate the skip trajectory forward in time by numerically integrating the full nonlinear equations of motion. Bank angle is adjusted in an iterative fashion until the predicted range matches the range-to-go. The predictor–corrector guidance schemes in [4,5] successfully produce skip trajectories with maximum downrange distances of 5400 n mile (90-deg range angle) and 4300 n mile (72-deg range angle), respectively.

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This Note presents a guidance method based on analytical entry trajectories. The skip-entry phase is predicted by using closed-form expressions that are derived from a matched asymptotic expansion analysis [6–10]. Trajectory planning for the atmospheric skip is accomplished by iterating on the vertical L/D component λ until the desired downrange distance is produced. Numerical results are presented to demonstrate the effectiveness of the guidance method.

System Models

Equations of Motion

The governing equations for a point-mass spacecraft in an Earth-centered Cartesian inertial frame are

$$\dot{\mathbf{r}} = \mathbf{v} \tag{1}$$

$$\dot{\mathbf{v}} = \mathbf{a}_g + \mathbf{a}_a \tag{2}$$

where $\mathbf{r}$ is the inertial position vector, $\mathbf{v}$ is the inertial velocity vector, $\mathbf{a}_g$ is the acceleration due to gravity, and $\mathbf{a}_a$ is the acceleration due to aerodynamic forces. Gravitational acceleration is computed by using an inverse-square model; total aerodynamic acceleration is the sum of lift acceleration $\mathbf{a}_L$ and drag acceleration $\mathbf{a}_D$, and their respective magnitudes are

$$a_L = \frac{\bar{q}SC_L}{m} \qquad a_D = \frac{\bar{q}SC_D}{m} \tag{3}$$

where C_L and C_D are lift and drag coefficients, S is the reference area, m is the spacecraft mass, and dynamic pressure is $\bar{q} = \rho V_{\mathrm{rel}}^2/2$. Relative velocity V_{rel} (or airspeed) is computed by subtracting the inertial velocity of the atmosphere from the inertial velocity of the vehicle (it is assumed that the atmosphere rotates with the Earth). Atmospheric density ρ is computed using the standard atmosphere.

Vehicle Models

The Apollo Command Module (CM) is used for guidance algorithm development, and the appropriate vehicle data is taken from [1]. The reference area is $S = 129.9\ \mathrm{ft}^2$ and mass is $12,125\ \mathrm{lb_m}$ (376.9 slugs). The CM is a symmetric body with an offset center of gravity, which causes the CM to trim aerodynamically at angles of attack ranging from 153 to 167 deg. Lift and drag coefficients are computed by linearly interpolating the trim aerodynamic coefficients with Mach number M as the independent variable. Lift-to-drag ratio L/D ranges from 0.30 (hypersonic flight) to a maximum value of 0.43 at $M = 1.65$.

Skip-Entry Guidance

Analytical Skip Trajectories Using Matched Asymptotic Expansions

The proposed guidance method uses analytical trajectory solutions to predict and plan the skip-entry profile. Shi and Pottsepp [6], Shi et al. [7], Vinh et al. [8], and Calise and Melamed [9] developed closed-form solutions for atmospheric skip-entry trajectories using the method of matched asymptotic expansions. Mease and McCreary [10] generalized the solutions from [6,7] and proposed guidance strategies for an aeroassist maneuver. A summary of the matched asymptotic expansion method is presented here; the reader can find detailed discussion in [10]. The closed-form solutions are in terms of nondimensional velocity v and altitude h:

$$v = \frac{V}{(\mu/r_{\mathrm{ref}})^{1/2}} \qquad h = \frac{r - r_{\mathrm{ref}}}{r_{\mathrm{ref}}} \tag{4}$$

where V is (dimensional) velocity, r is the radial distance from the center of the Earth to the vehicle, r_{ref} is a reference radius, and μ is the gravitational parameter. Other dimensionless parameters are

$$b = \frac{HC_D\rho_{\mathrm{ref}}}{(m/S)} \qquad \varepsilon = H/r_{\mathrm{ref}} \tag{5}$$

Atmospheric density ρ is computed using an exponential model:

$$\rho = \rho_{\mathrm{ref}} \exp[-(r - r_{\mathrm{ref}})/H] \tag{6}$$

where $H = 25{,}919$ ft is the scale height of the density model and $\rho_{\mathrm{ref}} = 5.3649(10^{-7})$ slugs/ft^3 is the density at the reference radius. The local density model parameters were selected to fit the standard atmosphere at an altitude of 196,850 ft (60 km), which is near the pull-up altitude of the skip maneuver. The composite solution for an atmospheric skip trajectory [10] is

$$v^2 = \frac{-2h}{1+h} + v_*^2 \exp\left[\frac{-2(\tilde{\gamma} - \tilde{\gamma}_*)}{\lambda}\right] \tag{7}$$

$$\cos\gamma = v_* \cos\gamma_* [v_*^2(1+h)^2 - 2h(1+h)]^{-1/2} + \frac{\lambda b}{2}\exp\left[\frac{-h}{\varepsilon}\right] \tag{8}$$

Equations (7) and (8) are closed-form solutions for velocity and flight-path angle γ as a function of altitude h during a skip entry. Clearly, these solutions depend on vehicle parameters, the vertical component of lift-to-drag ratio $\lambda = (C_L/C_D)\cos\sigma$ (σ is the bank angle), and the exponential model for density. The closed-form solution requires the following composite constants for velocity and flight-path angle:

$$v_*^2 = v_i^2 + \frac{2h_i}{1+h_i} \tag{9}$$

$$\cos\gamma_* = [(1+h_i)v_i \cos\gamma_i]/v_* \tag{10}$$

where the subscript i denotes the initial conditions at EI. Auxiliary flight-path angle $\tilde{\gamma}$ is determined by

$$\cos\tilde{\gamma} = \cos\tilde{\gamma}_* + \frac{\lambda b}{2}\exp[-h/\varepsilon] \tag{11}$$

The composite flight-path angle constant $\tilde{\gamma}_*$ is determined from Eq. (11) by using the minimum altitude of the skip trajectory ($h_{\min}$) and $\tilde{\gamma} = 0$:

$$\cos\tilde{\gamma}_* = 1 - \frac{\lambda b}{2}\exp[-h_{\min}/\varepsilon] \tag{12}$$

Once constants v_* and γ_* have been determined from Eqs. (9) and (10) and the initial conditions, the minimum altitude of the skip trajectory is determined from Eq. (8) and the condition $\cos\gamma = 1$. Equation (8) is transcendental in altitude, and therefore a Newton iteration scheme is used to determine $h_{\min}$. Fortunately, the analytical derivative of Eq. (8) with respect to h is available, and the subsequent Newton iteration converges in four–five iterations.

Skip Trajectory Planning

Similar to Apollo entry guidance, trajectory planning for the atmospheric skip phase is the foundation of the proposed guidance algorithm. Figure 1 shows the various guidance phases of a skip-entry trajectory for a downrange distance of about 4500 n mile. The "down" phase starts at EI and ends at minimum altitude for the skip entry (zero flight-path angle), the "up" phase starts at minimum altitude and ends at atmospheric exit, the Kepler phase ends at the second atmospheric entry point, and the final entry phase takes the

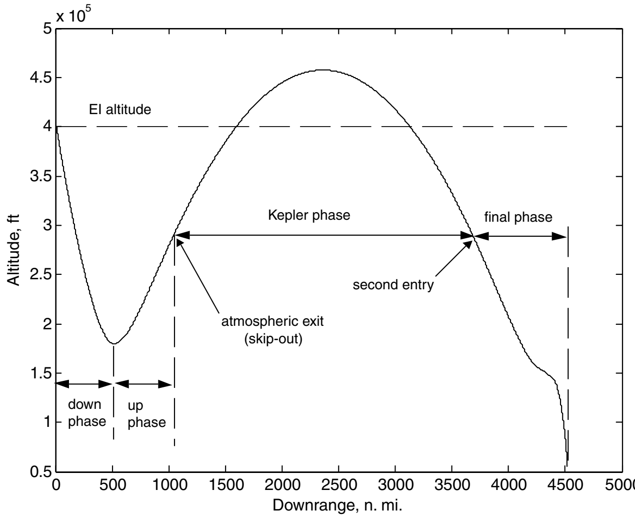

Fig. 1 Skip-entry profile and guidance phases.

vehicle from the second entry to parachute deployment for landing. Note that the altitude boundary for the Kepler phase is not necessarily EI, but rather an altitude at which drag acceleration is sufficiently diminished (Apollo guidance defines skip-out as the point at which drag acceleration is less than 0.2 g). Because the matched asymptotic expansion method uses altitude as the independent variable, it is the trigger for starting the Kepler phase. The Kepler-phase altitude is fixed at 213,255 ft (65 km) for downrange distances less than 2500 n mile and 278,871 ft (85 km) for skip trajectories for which downrange distance exceeds 2500 n mile. The accuracy of predicting long-range skip trajectories is improved by using higher exit altitudes because drag effects are diminished.

The proposed skip-entry guidance iterates on λ such that the predicted total downrange distance matches range-to-go to the target. At the initiation of entry (EI), an atmospheric skip trajectory is computed analytically by using Eqs. (7) and (8) for a nominal trial value for λ. The predicted total downrange distance is

$$R_{\mathrm{total}} = R_{\mathrm{down}} + R_{\mathrm{up}} + R_{\mathrm{Kepler}} + R_{\mathrm{final}} \tag{13}$$

Range angles for the down and up phases of the skip entry are estimated by integrating the derivative of range angle θ with respect to velocity:

$$\frac{\mathrm{d}\theta}{\mathrm{d}V} = \frac{\dot{\theta}}{\dot{V}} \qquad \dot{\theta} = \frac{V\cos\gamma}{r} \qquad \dot{V} = -a_D - \frac{\mu}{r^2}\sin\gamma \tag{14}$$

Discrete values of $\mathrm{d}\theta/\mathrm{d}V$ are computed by evaluating the analytical skip solutions [Eqs. (7) and (8)] at altitudes between h_{EI} and $h_{\min}$ (down phase) and $h_{\min}$ and h_{exit} (up phase), and trapezoidal-rule integration is used to integrate $\mathrm{d}\theta/\mathrm{d}V$. The respective downrange distances are then computed by multiplying the range angles with the Earth radius (i.e., $R_{\mathrm{down}} = r_E\theta_{\mathrm{down}}$ and $R_{\mathrm{up}} = r_E\theta_{\mathrm{up}}$). It should be noted that the up- and down-phase range angles can be computed using purely analytical expressions (see [3] for Apollo range calculations), and these analytical calculations rely on predictions for velocity, flight-path angle, and drag acceleration at $h_{\min}$ and h_{exit}.

The Kepler-phase range angle is computed by assuming two-body motion [11]:

$$\theta_{\mathrm{Kepler}} = 2\cos^{-1}\left[\frac{1 - Q\cos^2\gamma_{\mathrm{exit}}}{\sqrt{1 + Q(Q-2)\cos^2\gamma_{\mathrm{exit}}}}\right] \tag{15}$$

where $Q = V_{\mathrm{exit}}^2/(\mu/r_{\mathrm{exit}})$ is the ratio of the squares of predicted skip-out speed to local circular orbital speed. Skip-out speed V_{exit} and flight-path angle γ_{exit} are computed from the matched asymptotic expansion solutions (7) and (8) at skip-out altitude h_{exit}.

The final phase downrange R_{final} is computed by linearly interpolating a stored reference trajectory. The nominal final-phase profile for the Apollo guidance is used here, which was generated using a constant vertical L/D component $\lambda_{\text{final}} = 0.18$ ($\sigma = 53$ deg). The nominal reference trajectory uses velocity as the independent variable, and therefore $R_{\text{final}} = f(V_{\text{exit}})$, because velocity at the second entry is assumed to be equal to atmospheric exit velocity.

The actual range-to-go R_{go} is computed using the current position vector of the CM and the position vector of the targeted landing site. The Earth-rotation effect on the target's position vector is included by using an estimate for the remaining flight time (see [3] for details). Vertical-L/D component λ is adjusted using a secant iteration method until the predicted downrange error $|R_{\text{total}} - R_{\text{go}}| \leq$ 25 n mile. This iterative procedure is performed twice: initially at EI and again at the minimum altitude $h_{\min}$. Recalculating the remaining up phase of the skip trajectory at $h_{\min}$ greatly improves the predicted atmospheric exit states V_{exit} and γ_{exit}, because the composite constants υ_*, γ_*, and $\tilde{\gamma}_*$ for the matched asymptotic expansion solution can be recalculated using the vehicle's current state at minimum altitude. Our numerical results show that the iterative solution for λ converges in three–five iterations in all cases attempted (note that the Apollo guidance also used a secant iteration method that was limited to a total of seven iterations).

Bank-Angle Command

The open-loop value of λ from the analytical range-prediction iteration is augmented with closed-loop terms that use the Apollo feedback strategy [3]. During the down or up phase, the commanded vertical L/D component is

$$\lambda_C = \lambda_{\text{ref}} + K_V(V_{\text{ref}} - V) + K_{\text{RD}}(\dot{r}_{\text{ref}} - \dot{r}) \tag{16}$$

where the constant λ_{ref} is the result from the range-prediction iteration, and reference velocity V_{ref} and reference altitude rate $\dot{r}_{\text{ref}}$ are computed from the closed-form solutions (7) and (8) as a function of the current altitude. Feedback gains K_V and K_{RD} are based on the Apollo guidance gains; velocity gain K_V (in seconds per foot) is proportional to the current drag acceleration a_D:

$$K_V = 0.0034 \frac{a_D - a_D^{\text{exit}}}{a_D^{h\min} - a_D^{\text{exit}}} \tag{17}$$

where $a_D^{\text{exit}} = 4$ ft/s^2 is the approximate drag acceleration near skip-out altitude, and $a_D^{h\min}$ is the estimated drag acceleration at the pull-up altitude. The altitude-rate gain K_{RD} (in seconds per foot) is

$$K_{\text{RD}} = 0.0116 \left(\frac{a_D - a_D^{\text{exit}}}{a_D^{h\min} - a_D^{\text{exit}}} \right)^2 \tag{18}$$

During the final phase (second entry), the commanded vertical L/D component is based on linear perturbation theory and the stored reference trajectory:

$$\lambda_C = \lambda_{\text{final}} + \frac{\partial (L/D)}{\partial R}(R_{\text{go}} - R_{\text{pred}}) \tag{19}$$

where the predicted range is

$$R_{\text{pred}} = R_{\text{final}} + \frac{\partial R}{\partial \dot{r}}(\dot{r} - \dot{r}_{\text{ref}}) + \frac{\partial R}{\partial a_D}(a_D - a_{D\text{ref}}) \tag{20}$$

All reference values are computed by linearly interpolating the stored reference profile with velocity as the independent variable. The partial derivatives are computed using the method of adjoints and a linearized system, and they are also stored as a function of velocity (see [3] for details).

The bank angle is

$$\sigma = \cos^{-1}\left(\frac{\lambda_C}{L/D_{\max}} \right) \tag{21}$$

where $L/D_{\max} = 0.3$. The Apollo lateral guidance logic is used here to command bank reversals if the cross-range error exceeds a dead-zone value, where the dead-zone threshold is proportional to V^2 to limit the number of reversals (see [3] for details).

Numerical Results

Several skip-entry trajectories are computed to demonstrate the proposed guidance method. Simulink® is used to numerically integrate a nondimensional form of the governing equations of motion (1) and (2), and a third-order Bogacki–Shampine routine is employed with a fixed step size of 0.2 s. Nominal initial conditions at EI are based on Apollo 10 data from [1]: inertial velocity is 36,309.3 ft/s, inertial flight-path angle is −6.616 deg, and altitude is 406,441.3 ft. No initial heading error is assumed for the great circle connecting the EI and target-position vectors. The simulation is terminated when relative velocity (airspeed) becomes subsonic (parachute deployment soon follows). Terminal cross-track position error is determined from the angular error between the final great-circle arc and the target position, and in-track error is computed from the difference between the actual range-to-go R_{go} and the predicted range R_{pred} in Eq. (20). Note that R_{final} (the reference range-to-go for the final phase) is about 3 n mile when relative velocity becomes subsonic.

Table 1 summarizes the performance of the proposed analytical guidance method for several initial range angles. In-track and cross-track errors are well below 1 n mile for both short (30-deg range angle) and long (170-deg range angle) skip entries. Columns 2 and 3 show the predicted skip-out states (V_{exit} and γ_{exit}) from the analytical skip calculations and range-prediction iteration, as well as the actual skip-out states. The closed-loop control law (16) adequately adjusts the vertical L/D such that errors at skip-out are relatively small. Column 4 presents the maximum altitude h_{skip} after the pull-up maneuver. Note that for a short-range entry ($\theta = 30$ deg), h_{skip} is less than EI altitude (∼400,000 ft), and for a 60-deg range angle, h_{skip} is essentially at EI altitude. Atmospheric exit states V_{exit} and γ_{exit} (and maximum skip altitude h_{skip}) all increase with range angle; note that V_{exit} for the 170-deg range-angle case is very close to the local circular speed (25,765 ft/s). For the 90-deg range-angle case, the Earth rotates more than 6 deg during the skip entry, and so the final downrange distance is over 5700 n mile. For the 170-deg range-angle case, Earth rotation is nearly 10 deg, and the downrange distance is over 10,800 n mile (halfway around the Earth). Maximum drag acceleration (which is essentially the maximum load factor, because the flight-path angle is shallow) is between 6–7 g, which is typical of the gravity loads experienced by the Apollo missions.

Table 2 summarizes the performance of the proposed guidance for cases that involve dispersions. The initial range angle is fixed at 90 deg for all cases. The proposed guidance method provides small

Table 1 Performance of proposed guidance method for various range angles

Initial range angle, deg	Predicted/actual V_{exit}, ft/s	Predicted/actual γ_{exit}, deg	Maximum h_{skip}, ft	Maximum a_D, g	In-track error, n mile	Cross-track error, n mile
30	22,730/22,584	1.70/2.09	273,160	7.39	0.56	0.00
60	24,424/24,438	2.41/2.35	395,615	6.49	0.51	−0.03
90	24,983/25,008	2.66/2.48	555,191	6.18	0.54	−0.01
120	25,305/25,338	2.81/2.58	719,279	6.04	0.51	0.26
150	25,532/25,569	2.95/2.68	935,462	5.98	0.54	−0.21
170	25,663/25,702	3.09/2.79	1,136,596	5.99	0.66	−0.05

Table 2 Guidance performance for 90-deg range angle with dispersions

Dispersion	Predicted/actual V_{exit}, ft/s	Predicted/actual γ_{exit}, deg	Maximum h_{skip}, ft	Maximum a_D, g	In-track error, n mile	Cross-track error, n mile
$\Delta\gamma_{\text{EI}} = +0.2$ deg	25,082/25,102	2.24/2.13	510,079	5.50	0.55	−0.04
$\Delta\gamma_{\text{EI}} = -0.2$ deg	24,853/24,892	3.23/2.95	617,365	6.97	0.49	0.23
$\Delta C_L = +5\%$, $\Delta C_D = -5\%$	25,056/25,095	2.38/2.15	513,904	6.10	0.54	0.05
$\Delta C_L = -5\%$, $\Delta C_D = +5\%$	24,898/24,922	2.99/2.88	611,112	6.27	0.64	−0.15
$\Delta\rho = +20\%$	25,050/25,034	2.40/2.78	625,520	6.67	0.29	−0.14
$\Delta\rho = -10\%$	24,943/24,993	2.82/2.31	519,390	6.69	0.87	−0.56

terminal errors (below 1 n mile) for cases that involve dispersions in initial flight-path angle at EI, L/D, and atmospheric density. Aerodynamic and density dispersions are applied as constant multipliers to the appropriate system models, and guidance is not aware of these dispersions. If density variations are increased to -15 and $+25\%$, then the resulting in-track terminal errors grow to approximately 76 n mile (undershoot) and -23 n mile. (overshoot), respectively (note that density variations in Table 2 are limited to -10 and $+20\%$). Loss of terminal accuracy is due to the guidance algorithm's reliance on a fixed-density model and a single fixed reference path for the final phase (second entry). It is likely that the proposed guidance method could be made more robust to density variations by using the measured drag acceleration during the down phase to adjust the exponential density model that is used in the analytical skip calculations. It should be noted that the proposed guidance can provide acceptable terminal accuracy for the dispersed cases with an initial range angle of 120 deg. However, there is no feasible trajectory for a steep entry ($\Delta\gamma_{\text{EI}} = -0.2$ deg) when the range angle is greater than 127 deg, because a "full lift-up" trajectory falls short of the target. The proposed guidance can also provide acceptable accuracy for a 150-deg range trajectory for the cases involving shallow entry, increased lift, and density variations between -10 and 20%.

Finally, several skip entries were obtained by using the original Apollo guidance from [3]. Acceptable terminal position errors are possible for range angles up to about 95 deg (nominal case with no dispersions). Recall that the maximum range of the Apollo guidance is 2500 n mile (41-deg range angle). The Apollo guidance method demonstrates significant loss of accuracy for range angles beyond 95 deg and was found to be more sensitive to dispersions in density when compared with the proposed guidance method.

Conclusions

A new guidance method was developed for the skip entry of a capsule with low-lift-to-drag L/D characteristics. The guidance algorithm uses analytical skip trajectory calculations to iteratively adjust the vertical L/D component until the predicted range to the target matches the actual range-to-go. Once the guidance has converged, the analytical skip-out trajectory is used as a reference path for closed-loop targeting of the desired velocity and flight-path angle at atmospheric exit. Numerical simulations show that the proposed guidance method can provide accurate skip trajectories for range angles as great as 180 deg (halfway around the Earth), which greatly exceeds the range capabilities of the Apollo guidance system. These results demonstrate that the proposed analytical guidance method may be a feasible candidate as either the primary or backup entry guidance system for a new crew-return vehicle.

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